Paper Reference(s)

**6666/01**

# Edexcel GCE

## Core Mathematics C4

## Bronze Level B5

## Time: 1 hour 30 minutes

| **Materials required for examination papers** | **Items included with question** |
| --- | --- |
| Mathematical Formulae (Green) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.**

### Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

**Suggested grade boundaries for this paper:**

| A* | A | B | C | D | E |
| --- | --- | --- | --- | --- | --- |
| 68 | 59 | 52 | 47 | 40 | 34 |

**1.**

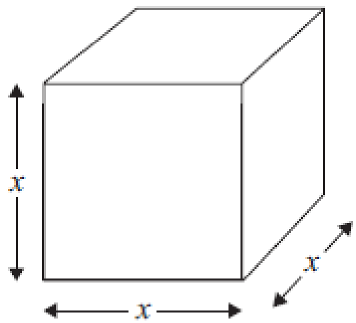



**Figure 1**

Figure 1 shows a metal cube which is expanding uniformly as it is heated.

At time $t$ seconds, the length of each edge of the cube is $x$ cm, and the volume of the cube is $V$ cm$^3$.

(*a*) Show that $\dfrac{dV}{dx} = 3x^2$.

**(1)**

Given that the volume, $V$ cm$^3$, increases at a constant rate of 0.048 cm$^3$ s$^{-1}$,

(*b*) find $\dfrac{dx}{dt}$ when $x = 8$,

**(2)**

(*c*) find the rate of increase of the total surface area of the cube, in cm$^2$ s$^{-1}$, when $x = 8$.

**(3)**

**June 2012**

---

**2.** (*a*) Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions.

**(3)**

(*b*) Hence find $\displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx$, where $x > 1$.

**(3)**

(*c*) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{dy}{dx} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$ . Give your answer in the form $y = f(x)$.

**(6)**

**January 2011**

---

**3.** (*a*) Use the binomial theorem to expand

$$(2 - 3x)^{-2}, \quad |x| < \frac{2}{3},$$

in ascending powers of $x$, up to and including the term in $x^3$. Give each coefficient as a simplified fraction.

**(5)**

$$f(x) = \frac{a + bx}{(2 - 3x)^2}, \quad |x| < \frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of f($x$), in ascending powers of $x$, the coefficient of $x$ is 0 and the coefficient of $x^2$ is $\frac{9}{16}$.

Find

(*b*) the value of *a* and the value of *b*,

**(5)**

(*c*) the coefficient of $x^3$ , giving your answer as a simplified fraction.

**(3)**

**January 2011**

---

**4.**

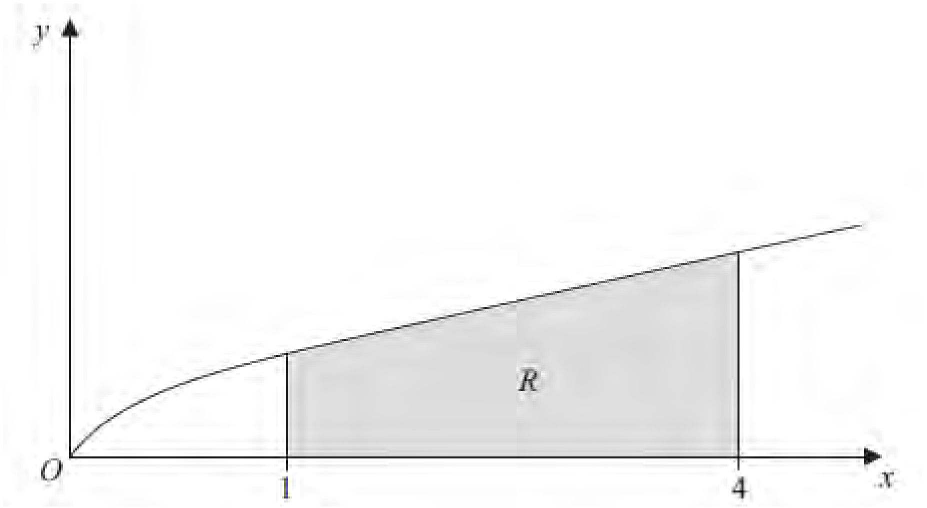



**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{x}{1+\sqrt{x}}$. The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(*a*) Copy and complete the table with the value of $y$ corresponding to $x = 3$, giving your answer to 4 decimal places.

**(1)**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 0.5 | 0.8284 | | 1.3333 |

(*b*) Use the trapezium rule, with all the values of $y$ in the completed table, to obtain an estimate of the area of the region $R$, giving your answer to 3 decimal places.

**(3)**

(*c*) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of $R$.

**(8)**

**January 2013**

**5.**

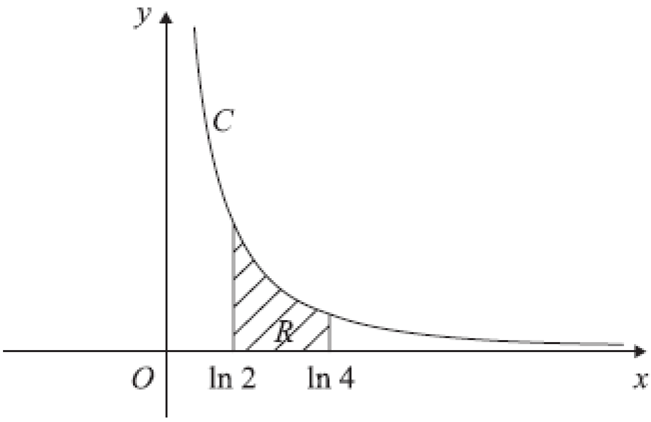



**Figure 3**

The curve $C$ has parametric equations

$$x = \ln(t+2), \quad y = \frac{1}{(t+1)}, \quad t > -1.$$

The finite region $R$ between the curve $C$ and the $x$-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

($a$) Show that the area of $R$ is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t\,.$$

**(4)**

($b$) Hence find an exact value for this area.

**(6)**

($c$) Find a cartesian equation of the curve $C$, in the form $y = \mathrm{f}(x)$.

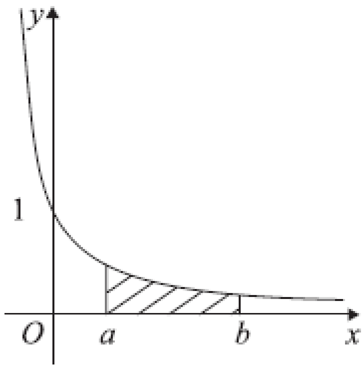
**(4)**

($d$) State the domain of values for $x$ for this curve.

**(1)**

**January 2008**

**6.**




The curve shown in Figure 2 has equation $y = \dfrac{1}{(2x+1)}$. The finite region bounded by the curve, the $x$-axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the $x$-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of $a$ and $b$.

**(5)**

**January 2008**

---

**7.** (*a*) Express $\dfrac{1}{P(5-P)}$ in partial fractions.

**(3)**

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{15}P(5-P), \qquad t \geq 0,$$

where $P$, in thousands, is the population of meerkats and $t$ is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(*b*) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + c\mathrm{e}^{-\frac{1}{3}t}}$$

where $a$, $b$ and $c$ are integers.

**(8)**

(*c*) Hence show that the population cannot exceed 5000.

**(1)**

**January 2012**

---

**TOTAL FOR PAPER: 75 MARKS**

| | | | |
|---|---|---|---|
| **1.** (a) | $V = x^3 \Rightarrow \dfrac{dV}{dx} = 3x^2$ * cso | B1 | **(1)** |
| (b) | $\dfrac{dx}{dt} = \dfrac{dx}{dV} \times \dfrac{dV}{dt} = \dfrac{0.048}{3x^2}$ | M1 | |
| | At $x = 8$, $\dfrac{dx}{dt} = \dfrac{0.048}{3\left(8^2\right)} = 0.00025 \quad \left(\text{cm s}^{-1}\right)$ $2.5 \times 10^{-4}$ | A1 | **(2)** |
| (c) | $S = 6x^2 \Rightarrow \dfrac{dS}{dx} = 12x$ | B1 | |
| | $\dfrac{dS}{dt} = \dfrac{dS}{dx} \times \dfrac{dx}{dt} = 12x\left(\dfrac{0.048}{3x^2}\right)$ | M1 | |
| | At $x = 8$ $\dfrac{dS}{dt} = 0.024 \quad \left(\text{cm}^2\ \text{s}^{-1}\right)$ | A1 | **(3)** |
| | | | **[6]** |

| Question Number | Scheme | Marks |
|---|---|---|
| **2.** **(a)** | $\dfrac{5}{(x-1)(3x+2)} = \dfrac{A}{x-1} + \dfrac{B}{3x+2}$ <br> $5 = A(3x+2) + B(x-1)$ <br> $x \to 1$ $\quad 5 = 5A \Rightarrow A = 1$ <br> $x \to -\dfrac{2}{3}$ $\quad 5 = -\dfrac{5}{3}B \Rightarrow B = -3$ | <br><br>M1 A1<br>A1 (3) |
| **(b)** | $\displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx = \int \left(\frac{1}{x-1} - \frac{3}{3x+2}\right) dx$ <br> $= \ln(x-1) - \ln(3x+2) \quad (+C)$ ft constants | <br>M1 A1ft A1ft<br>(3) |
| **(c)** | $\displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx = \int \left(\frac{1}{y}\right) dy$ <br> $\ln(x-1) - \ln(3x+2) = \ln y \quad (+C)$ <br> $y = \dfrac{K(x-1)}{3x+2}$ depends on first two Ms in (c) <br> Using $(2, 8)$ $\quad 8 = \dfrac{K}{8}$ depends on first two Ms in (c) <br> $y = \dfrac{64(x-1)}{3x+2}$ | M1<br>M1 A1<br>M1 dep<br>M1 dep<br>A1 (6)<br>**[12]** |

| Question Number | Scheme | Marks |
|---|---|---|
| **3.** | | |
| **(a)** | $(2-3x)^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$ | B1 |
| | $\left(1-\frac{3}{2}x\right)^{-2} = 1+(-2)\left(-\frac{3}{2}x\right)+\frac{-2.-3}{1.2}\left(-\frac{3}{2}x\right)^2+\frac{-2.-3.-4}{1.2.3}\left(-\frac{3}{2}x\right)^3+\ \ldots$ | M1 A1 |
| | $=1+3x+\frac{27}{4}x^2+\frac{27}{2}x^3+\ \ldots$ | |
| | $(2-3x)^{-2}=\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \ldots$ | M1 A1 (5) |
| **(b)** | $\mathrm{f}(x)=(a+bx)\left(\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \ldots\right)$ | |
| | Coefficient of $x$; $\frac{3a}{4}+\frac{b}{4}=0$ $(3a+b=0)$ | M1 |
| | Coefficient of $x^2$; $\frac{27a}{16}+\frac{3b}{4}=\frac{9}{16}$ $(9a+4b=3)$ A1 either correct | M1 A1 |
| | Leading to $a=-1,\ b=3$ | M1 A1 (5) |
| **(c)** | Coefficient of $x^3$ is $\frac{27a}{8}+\frac{27b}{16}=\frac{27}{8}\times(-1)+\frac{27}{16}\times 3$ | M1 A1ft |
| | $=\frac{27}{16}$ cao | A1 (3) |
| | | **[13]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **4.** (a) | 1.0981 | | B1 **cao** |
| | | | **[1]** |
| (b) | Area $\approx \frac{1}{2} \times 1 ; \times \underline{\left[0.5 + 2\left(0.8284 + \text{their } 1.0981\right) + 1.33\right.}$ | | B1; <br> $\underline{\text{M1}}$ |
| | $= \frac{1}{2} \times 5.6863 \;=\; 2.84315 \;=\; 2.843 \text{ (3 dp)}$ | 2.843 or awrt 2.843 | A1 |
| | | | **[3]** |
| (c) | $\left\{u = 1 + \sqrt{x}\right\} \Rightarrow \underline{\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}}$ or $\underline{\frac{\mathrm{d}x}{\mathrm{d}u} = 2(u-1)}$ | | $\underline{\text{B1}}$ |
| | $\left\{\int \frac{x}{1+\sqrt{x}}\,\mathrm{d}x = \right\} \int \frac{(u-1)^2}{u}.\,2(u-1)\;\mathrm{d}u$ | $\int \frac{(u-1)^2}{u}......$ | M1 |
| | | $\int \frac{(u-1)^2}{u}.\,2(u-1)$ | A1 |
| | $= 2\int \frac{(u-1)^3}{u}\,\mathrm{d}u \;=\{2\}\int \frac{(u^3 - 3u^2 + 3u}{u}$ | Expands to give a "four term" cubic in $u$. <br> Eg: $\pm Au^3 \pm Bu^2 \pm Cu \pm D$ | M1 |
| | $= \{2\}\int \left(u^2 - 3u + 3 - \frac{1}{u}\right)\mathrm{d}u$ | An attempt to divide at least three terms in ***their cubic*** by $u$. See notes. | M1 |
| | $= \{2\}\left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right)$ | $\int \frac{(u-1)^3}{u} \to \left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right)$ | A1 |
| | $\text{Area}(R) = \left[\frac{2u^3}{3} - 3u^2 + 6u - 2\ln u\right]_2^3$ | | |
| | $= \left(\frac{2(3)^3}{3} - 3(3)^2 + 6(3) - 2\ln 3\right) - \left(\frac{2(2)^3}{3} - 3(2)^2 + 6(2) - 2\right.$ | Applies limits of 3 and 2 in $u$ or 4 and 1 in $x$ and subtracts either way round. | M1 |
| | $= \frac{11}{3} + 2\ln 2 - 2\ln 3$ or $\frac{11}{3} + 2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3} - \ln\left(\frac{9}{4}\right)$, etc | Correct exact answer or equivalent. | A1 |
| | | | **[8]** |
| | | | **12** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **5.** (a) | $\left[x=\ln(t+2),\ \ y=\frac{1}{t+1}\right],\quad \Rightarrow \frac{dx}{dt}=\frac{1}{t+2}$ | Must state $\frac{dx}{dt}=\frac{1}{t+2}$ | B1 |
| | $\text{Area}(R)=\int_{\ln 2}^{\ln 4}\frac{1}{t+1}dx;=\int_0^2\left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)dt$ | Area $=\int\frac{1}{t+1}dx$. Ignore limits. | M1; |
| | | $\int\left(\frac{1}{t+1}\right)\times\left(\frac{1}{t+2}\right)dt$. Ignore limits. | A1 |
| | Changing limits, when:<br>$x=\ln 2\ \Rightarrow \ln 2=\ln(t+2)\ \Rightarrow\ 2=t+2\ \Rightarrow t=0$<br>$x=\ln 4\ \Rightarrow \ln 4=\ln(t+2)\ \Rightarrow\ 4=t+2\ \Rightarrow t=2$ | changes limits $x\to t$ so that $\ln 2\to 0$ and $\ln 4\to 2$ | B1 |
| | Hence, $\text{Area}(R)=\int_0^2\frac{1}{(t+1)(t+2)}dt$ | | **[4]** |
| (b) | $\left(\frac{1}{(t+1)(t+2)}\right)=\frac{A}{(t+1)}+\frac{B}{(t+2)}$ | $\frac{A}{(t+1)}+\frac{B}{(t+2)}$ with $A$ and $B$ found | M1 |
| | $1=A(t+2)+B(t+1)$ | | |
| | Let $t=-1,\quad 1=A(1)\quad\Rightarrow\ \underline{A=1}$<br>Let $t=-2,\quad 1=B(-1)\ \Rightarrow\ \underline{B=-1}$ | Finds both $A$ and $B$ correctly. Can be implied. (See note below) | A1 |
| | $\int_0^2\frac{1}{(t+1)(t+2)}dt=\int_0^2\frac{1}{(t+1)}-\frac{1}{(t+2)}dt$ | | |
| | $=\left[\ln(t+1)-\ln(t+2)\right]_0^2$ | Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$<br>Both ln terms correctly ft. | dM1<br>A1 $\sqrt{}$ |
| | $=(\ln 3-\ln 4)-(\ln 1-\ln 2)$ | Substitutes ***both*** limits of 2 and 0 and subtracts the correct way round. | ddM1 |
| | $=\ln 3-\ln 4+\ln 2=\ln 3-\ln 2=\ln\left(\frac{3}{2}\right)$ | $\underline{\ln 3-\ln 4+\ln 2}$ or $\underline{\ln\left(\frac{3}{4}\right)-\ln\left(\frac{1}{2}\right)}$ or $\underline{\ln 3-\ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$ (must deal with ln 1) | A1 aef isw **[6]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **6.** | Volume $= \underline{\pi \int_a^b \left(\frac{1}{2x+1}\right)^2 \mathrm{d}x} = \pi \int_a^b \frac{1}{(2x+1)^2} \mathrm{d}x$ | Use of $\underline{V = \pi \int y^2} \mathrm{d}x$. Can be implied. Ignore limits. | B1 |
| | $= \pi \int_a^b (2x+1)^{-2} \mathrm{d}x$ | | |
| | $= (\pi)\left[\frac{(2x+1)^{-1}}{(-1)(2)}\right]_a^b$ | | |
| | $= (\pi)\left[\underline{-\tfrac{1}{2}(2x+1)^{-1}}\right]_a^b$ | Integrating to give $\underline{\pm p(2x+1)^{-1}}$ | M1 |
| | | $\underline{-\tfrac{1}{2}(2x+1)^{-1}}$ | A1 |
| | $= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$ | Substitutes limits of $b$ and $a$ and subtracts the correct way round. | dM1 |
| | $= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$ | | |
| | $= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$ | | |
| | $= \frac{\pi(b-a)}{(2a+1)(2b+1)}$ | $\underline{\frac{\pi(b-a)}{(2a+1)(2b+1)}}$ | A1 aef |
| | | | **[5]** |
| | | | **5 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **7.** (a) | $1 = A(5-P) + BP$ | Can be implied. | M1 |
| | $A = \frac{1}{5},\ B = \frac{1}{5}$ | Either one. | A1 |
| | giving $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$ | | A1 **cao, aef** |
| | | | **[3]** |
| (b) | $\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int \frac{1}{15}\,\mathrm{d}t$ | | B1 |
| | $\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \quad (+c)$ | | M1* A1ft |
| | $\{t=0,\ P=1 \Rightarrow\}\quad \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c$ | | dM1* |
| | $\left\{\Rightarrow\quad c = -\frac{1}{5}\ln 4\right\}$ | | |
| | eg: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$ | Using any of the subtraction (or addition) laws for logarithms CORRECTLY | dM1* |
| | $\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$ | | |
| | eg: $\frac{4P}{5-P} = \mathrm{e}^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = \mathrm{e}^{-\frac{1}{3}t}$ | Eliminate ln's correctly. | dM1* |
| | gives $4P = 5\mathrm{e}^{\frac{1}{3}t} - P\mathrm{e}^{\frac{1}{3}t} \Rightarrow P(4+\mathrm{e}^{\frac{1}{3}t}) = 5\mathrm{e}^{\frac{1}{3}t}$ | | |
| | $P = \frac{5\mathrm{e}^{\frac{1}{3}t}}{(4+\mathrm{e}^{\frac{1}{3}t})} \quad \left\{\frac{(\div\,\mathrm{e}^{\frac{1}{3}t})}{(\div\,\mathrm{e}^{\frac{1}{3}t})}\right\}$ | Make $P$ the subject. | dM1* |
| | $P = \frac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ or $P = \frac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$ etc. | | A1 |
| | | | **[8]** |
| (c) | $1 + 4\mathrm{e}^{-\frac{1}{3}t} > 1 \Rightarrow P < 5$. So population cannot exceed 5000. | | B1 |
| | | | **[1]** **(12 marks)** |

**Question 1**

This question was well done and more than half of the candidates gained full marks. The great majority of candidates gained full marks in parts (a) and (b). In part (c), most identified that they needed to find $\frac{\mathrm{d}A}{\mathrm{d}x}$ but some failed to realise that a cube has six faces and worked with $\frac{\mathrm{d}A}{\mathrm{d}x} = 2x$ rather than $12x$. Not all realised that a chain rule was now needed and some simply calculated the value of $\frac{\mathrm{d}A}{\mathrm{d}x}$ at $x = 8$. However, the majority were able to put together a meaningful chain rule and complete the question.

**Question 2**

Partial fractions are well understood and part (a) was usually fully correct. The majority used substitution to find the constants and comparing constants was rare, as was the use of the cover up rule. One error that was seen from time to time was $5A = 5 \Rightarrow A = 5$. Part (b) was also well done and the common error $\int \frac{3}{3x+2}\,\mathrm{d}x = 3\ln(3x+2)$ was seen less often than in some recent examinations.

Part (c) proved more difficult. Many could not separate the variables correctly and some did not even realise that this was necessary. Some kept the 5 with the *y* and this caused problems in applying the result of part (b) correctly. Those who established the appropriate method usually included a constant of integration and were able to obtain an equation to find its value. Making *y* the subject of the formula proved difficult and moving from an expression of the form $\ln y = \ln(\mathrm{f}(x)) + \ln k$ to $y = \mathrm{f}(x) + k$ was a common error.

**Question 3**

Part (a) was well done. Those who could write $(2-3x)^{-2}$ as $2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$ nearly always expanded correctly and those who could not were usually able to gain 2 or 3 marks by showing that they knew how to expand binomial expressions. The distribution of marks for part (b) was bimodal; the majority of candidates obtaining either 0 or 5 marks. Those who knew how to proceed were able to obtain two linear equations by comparing coefficients and solve them for *a* and *b*. Apart from occasional algebraic slips, these candidates usually obtained full marks. Those who did not know the appropriate method often gave up very quickly and, wisely, went on to the next question. Those who were able to solve part (b) almost always completed the question.

**Question 4**

This question was answered well across all abilities. Whilst the majority of candidates were able to score full marks in Q4(a) and Q4(b), Q4(c) was found to be more challenging. Despite the challenging nature of Q4(c), it was encouraging to see a good number of clear, logical and accurate solutions.

In Q4(a), although most candidates correctly computed 1.0981, a significant number wrote 1.0980, suggesting that truncation rather than rounding was applied by some at this stage.

In applying the trapezium rule in Q4(b), a small minority of candidates multiplied $\frac{1}{2}$ by $\frac{3}{4}$ instead of $\frac{1}{2}$ by 1. Whilst the table of values shows clearly an interval width of 1, the application of a formula $h = \frac{b-a}{n}$ with $n = 4$ instead of $n = 3$ sometimes caused this error. Other errors included the occasional bracketing mistake and the occasional calculation error following a correctly written expression.

In Q4(c), candidates clearly knew that they needed to transform an integral in $\mathrm{f}(x)$ into an integral in $\mathrm{g}(u)$ and most began as expected by finding $\frac{\mathrm{d}u}{\mathrm{d}x}$. The omission of $\frac{\mathrm{d}x}{\mathrm{d}u}$ from the integral expression for the area resulted in a simpler function in $u$ with a consequent loss of marks. Other errors on substitution were the use of $u^2 - 1$ instead of $(u-1)^2$ and multiplication by $\frac{\mathrm{d}u}{\mathrm{d}x}$ instead of by $\frac{\mathrm{d}x}{\mathrm{d}u}$. For those who did obtain an expression of the form $\frac{2(u-1)^3}{u}$, many expanded the cubic part, divided the result by $u$ and integrated the result as expected, although sometimes making an error on expansion or forgetting to multiply by the 2. Some candidates, however, attempted integration by parts or integration without expansion on $\frac{(u-1)^3}{u}$ resulting in erroneous expressions such as $(u-1)^4 \ln u$ or $\frac{(u-1)^4}{4u}$. Despite previous errors, the majority of candidates were able to apply the changed limits of 3 and 2 appropriately to an ‘integrated’ function in $u$. A return to $x$ limits would have been acceptable but was seldom seen and only occasionally $x$ limits were used erroneously in a function in $u$.

**Question 5**

The majority of candidates were able to show the integral required in part (a). Some candidates, however, did not show evidence of converting the given limits, whilst for other candidates this was the only thing they were able to do.

In part (b), it was disappointing to see that some strong candidates were unable to gain any marks by failing to recognise the need to use partial fractions. Those candidates who split the integral up as partial fractions usually gained all six marks, while those candidates who failed to use partial fractions usually gave answers such as $\ln(t^2 + 3t + 2)$ or $\ln(t+1) \times \ln(t+2)$ after integration. A few candidates only substituted the limit of 2, assuming that the result of substituting a limit of 0 would be zero Few candidates gave a decimal answer instead of the exact value required by the question.

Part (c) was well answered by candidates of all abilities with candidates using a variety of methods as identified in the mark scheme. Occasionally some candidates were able to eliminate $t$ but then failed to make $y$ the subject.

The domain was not so well understood in part (d), with a significant number of candidates failing to correctly identify it.

**Question 6**

Most candidates used the correct volume formula to obtain an expression in terms of $x$ for integration. At this stage errors included candidates using either incorrect formulae of $\pi\int y\ \mathrm{d}x$, $2\pi\int y^2\ \mathrm{d}x$ or $\int y^2\ \mathrm{d}x$. Many candidates realised that they needed to integrate an expression of the form $(2x+1)^{-2}$ (or equivalent). The majority of these candidates were able to complete the integration correctly or at least achieve an integrated expression of the form $p(1+2x)^{-1}$. A few candidates, however, integrated to give an expression in terms of natural logarithms. A significant minority of candidates substituted the limits of $b$ and $a$ into their integrand the wrong way round. Only a minority of candidates were able to combine together their rational fractions to give an answer as a single simplified fraction as required by the question.

**Question 7**

In part (a), the majority of candidates were able to write down the correct identity to find their constants correctly, although a few candidates forgot to express their answer as a partial fraction as requested in the question.

A significant minority of candidates who completed part (a) correctly, made no attempt at part (b). Most candidates, however, were able to separate the variables, although some did this incorrectly, or did not try. The majority used their part (a) answer and integrated this to obtain an expression involving two ln terms. Although many integrated their expression correctly, some made a sign error by integrating $\frac{1}{5-P}$ to obtain $\ln(5-P)$. Those candidates who integrated $\frac{1}{5P}$ and $\frac{1}{25-5P}$ to $\frac{1}{5}\ln 5P$ and $-\frac{1}{5}\ln\left(25-5P\right)$, respectively, tended to find subsequent manipulation more difficult. A significant number of candidates did not attempt to find a constant of integration – with some omitting it from their working whilst others referring to "$+\ c$" and not attempting to use the boundary condition of $t=0$ and $P=1$ to find its value. Most candidates were able to apply the subtraction (or sometimes the addition) law of logarithms correctly for their expression but a number of candidates struggled to correctly remove the logarithms from their integrated equation, with incorrect manipulation of $\ln\left(\frac{P}{5-P}\right) = \frac{1}{3}t - \ln 4$ leading to $\frac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} - 4$ usually seen.

A significant number of those candidates who removed logarithms correctly were able to manipulate their result to make $P$ the subject of their equation, although a number of these candidates could not make the final step of manipulating $P = \frac{5\mathrm{e}^{\frac{1}{3}t}}{(4+\mathrm{e}^{\frac{1}{3}t})}$ into $P = \frac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$. Of all the 8 questions, question 8(b) was the most demanding in terms of a need for accuracy, and about 20% of candidates were able to score all 8 marks in this part.

Very few candidates gained the mark in part (c). Many were able to show that $P$ could not be equal to 5, and some of them also looked at what happens to $P$ as $t$ approaches infinity, but then failed to point out that the function was strictly increasing. Few candidates were able to state $1+4\mathrm{e}^{-\frac{1}{3}t} > 1$ implied $P < 5$, but some of them did not go on to give a conclusion in relation to the context of the question.

# Statistics for C4 Practice Paper Bronze Level B5

| | | | | Mean score for students achieving grade: | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Qu** | **Max score** | **Modal score** | **Mean %** | **ALL** | **A*** | **A** | **B** | **C** | **D** | **E** | **U** |
| **1** | 6 | | 75 | 4.52 | 5.86 | 5.49 | 4.91 | 3.93 | 2.85 | 1.97 | 1.15 |
| **2** | 12 | | 76 | 9.14 | 11.85 | 10.43 | 8.80 | 6.79 | 5.19 | 4.49 | 2.67 |
| **3** | 13 | | 73 | 9.52 | 12.83 | 11.40 | 8.92 | 6.68 | 4.97 | 4.03 | 1.88 |
| **4** | 12 | 12 | 69 | 8.31 | 11.63 | 9.56 | 7.75 | 6.10 | 5.06 | 3.94 | 2.52 |
| **5** | 15 | | 63 | 9.46 | | 12.46 | 9.00 | 6.44 | 4.78 | 2.32 | 1.44 |
| **6** | 5 | | 59 | 2.93 | | 3.89 | 2.93 | 1.93 | 1.68 | 0.89 | 0.47 |
| **7** | 12 | | 56 | 6.71 | 10.67 | 8.29 | 5.63 | 3.92 | 2.60 | 2.28 | 1.08 |
| | **75** | | **67** | **50.59** | | **61.52** | **47.94** | **35.79** | **27.13** | **19.92** | **11.21** |